7

Bipolar Junction Transistor (BJT)

- A three-terminal device that uses the voltage of the two terminals to control the current flowing in the third terminal.
  - The basis for amplifier design.
  - The basis for switch design.
  - The basic element of high speed integrated digital and analog circuits.
- Applications
  - Discrete-circuit design.
  - Analog circuits.
    - High frequency application such as radio frequency analog circuit.
  - Digital circuits.
    - High speed digital circuit such as emitter coupled circuit (ECC).
    - Bi-CMOS (Bipolar+CMOS) circuits that combines the advantages of MOSFET and bipolar transistors.
      - MOSFET: high-input impedance and low-power.
      - Bipolar transistors: high-frequency-operation and high-current-driving capabilities.
- Circuit symbol
  - The arrowhead on the emitter implies the polarity of the emitter-base voltage.
    - NPN: $v_{BE} > 0$.
    - PNP: $v_{EB} > 0$.

7.1 Structure

7.1.1 NPN Transistor

- Figure 7.2 depicts a simplified NPN transistor.
  - Emitter (E): heavily doped n-type region.
  - Base (B): lightly doped p-type region.
  - Collector (C): heavily doped n-type region.
  - Two diodes connected in series with opposite directions.
    - EBJ: Emitter-Base junction.

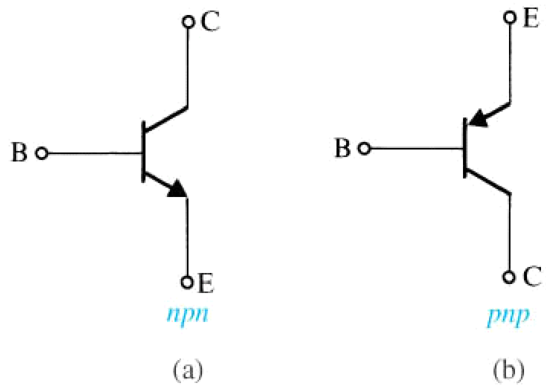

Figure 7.1: Circuit symbols of (a) NPN and (b) PNP transistors.

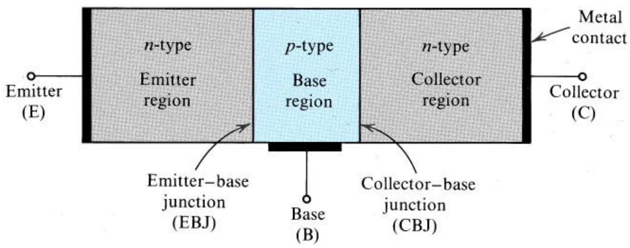

Figure 7.2: A simplified structure of the NPN transistor.

  * CBJ: Collector-Base junction.
- Figure 7.3 shows the cross-section view of an NPN transistor.
  - The NPN transistor has asymmetrical structure.
  - α and β parameters are different for forward active and reverse active modes.
- Modes of operations
  - Cutoff
    * EBJ (Reverse), CBJ (Reverse)
    * $v_{BE} < 0$, $v_{CB} > 0$.
  - Active (refer to Figure 7.7)
    * EBJ (Forward), CBJ (Reverse)
    * $v_{BE} > 0$, $v_{CB} > 0$.
  - Reverse Active
    * EBJ (Reverse), CBJ (Forward)
    * $v_{BE} < 0$, $v_{CB} < 0$.
  - Saturation
    * EBJ (Forward), CBJ (Forward)
    * $v_{BE} < 0$, $v_{CB} < 0$.

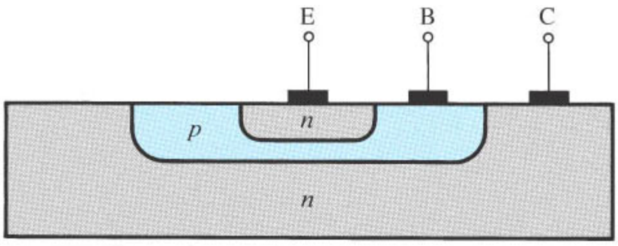

Figure 7.3: Cross-section of an NPN BJT.

- Figure 7.4 shows the voltage polarities and current flow in the NPN transistor biased in the active mode.

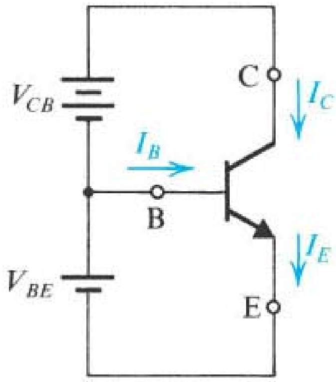

Figure 7.4: Voltage polarities and current flow in the NPN transistor biased in the active mode.

7.1.2 PNP Transistor

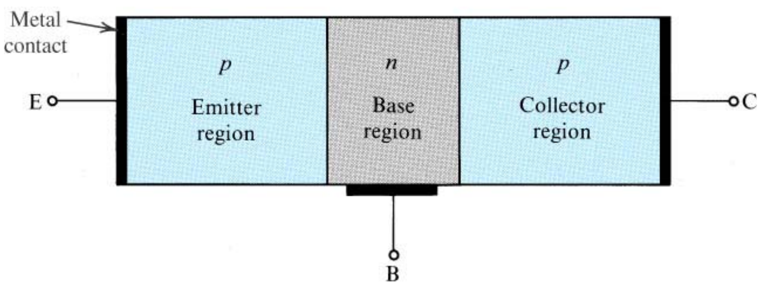

Figure 7.5: A simplified structure of the PNP transistor.

- Figure 7.5 depicts a simplified PNP transistor.
  - Emitter (E): heavily doped p-type region.

  - Base (B): lightly doped n-type region.
  - Collector (C): heavily doped p-type region.
  - Two diodes connected in series with opposite directions.
    * EBJ: Emitter-Base junction.
    * CBJ: Collector-Base junction.

- Modes of operations
  - Cutoff
    * EBJ (Reverse), CBJ (Reverse)
    * $v_{EB} < 0$, $v_{BC} < 0$.
  - Active (refer to Figure 7.7)
    * EBJ (Forward), CBJ (Reverse)
    * $v_{EB} > 0$, $v_{BC} > 0$.
  - Reverse Active
    * EBJ (Reverse), CBJ (Forward)
    * $v_{EB} < 0$, $v_{BC} < 0$.
  - Saturation
    * EBJ (Forward), CBJ (Forward)
    * $v_{EB} > 0$, $v_{CB} > 0$.
- Figure 7.6 shows the voltage polarities and current flow in the PNP transistor biased in the active mode.

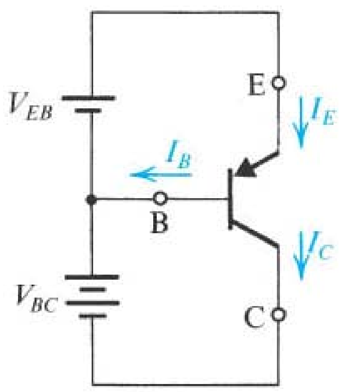

Figure 7.6: Voltage polarities and current flow in the PNP transistor biased in the active mode.

7.2 Operations of NPN Transistor

7.2.1 Active Mode

- Emitter-Base Junction

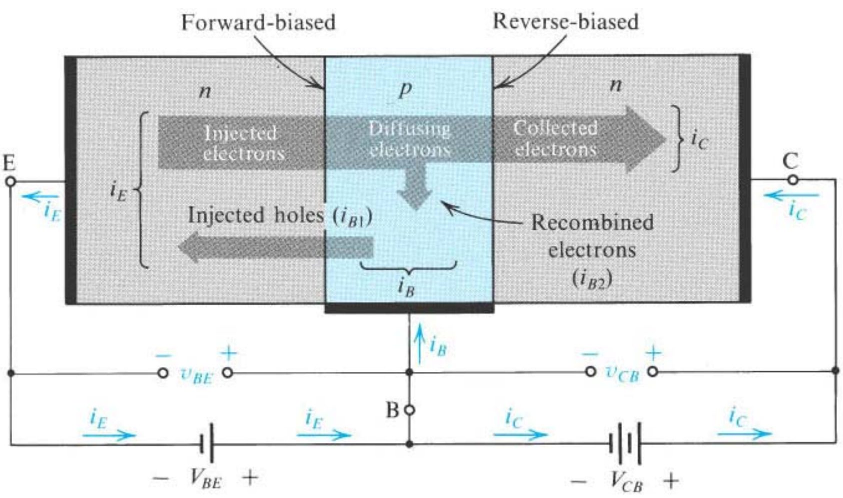

Figure 7.7: Current flow in an NPN transistor to operate in the active mode.

  - Forward bias, $v_{BE} > 0$.
  - Electrons in the emitter region are injected into the base causing a current i_{E1}.
  - Holes in the base region are injected into the emitter region causing a current i_{E2}.
    * Generally, $i_{E1} >> i_{E2}$.

$$i_E(t) = i_{E1} + i_{E2} \tag{7.1}$$

- Base region
  - Figure 7.8 depicts the concentration of minority carriers (electrons) in the base region.
  - Tapered concentration causes the electrons to diffuse through the base region toward the collector.
    * Some of the electrons may combine with the holes causing a concave shape of the profile.
    * The recombination process is quite small due to lightly doped and thin base region.

$$n_p(0) = n_{p0}e^{v_{BE}/V_T} \tag{7.2}$$

  - Diffusion current I_n (flowing from right to the left) is proportional to the slope of the concentration profile.
    * A_E is the cross-sectional area of the base-emitter junction.
    * D_n is the electron diffusivity in the base region.
    * W is the effective width of the base.

$$I_n = A_E q D_n \frac{dn_p(x)}{dx} = -A_E q D_n \frac{n_p(0)}{W} \tag{7.3}$$

- Collector-Base Junction
  - Reverse bias, $v_{BC} > 0$.
  - The electrons near the collector side are swept into the collector region causing zero concentration at the collector side.

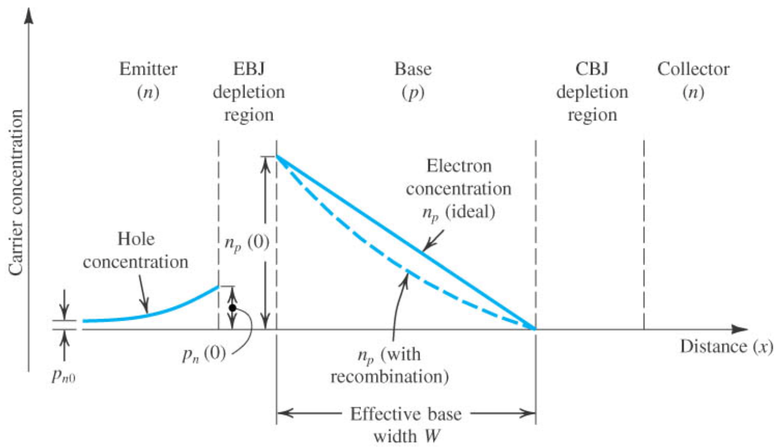

Figure 7.8: Profiles of minority carrier concentrations in the base and in the emitter of an NPN transistor.

- Collector current, i_C.
  - Most of the diffusing electrons will reach the collector region, i.e., $i_C = -I_n$.
    - Only a very small percentage of electrons are recombined with the holes in the base region.
  - As long as $v_{CB} > 0$, i_C is independent of v_{CB}.
    - The electrons that reach the collector side of the base region will be swept into the collector as collector current.

$$
\begin{aligned}
i_C &= -I_n \\
&= A_E q D_n \frac{n_p(0)}{W} \\
&= \frac{A_E q D_n n_{p0}}{W} e^{v_{BE}/V_T} \\
&= \frac{A_E q D_n n_i^2}{W N_A} e^{v_{BE}/V_T} \\
&= I_S e^{v_{BE}/V_T}
\end{aligned}
\tag{7.4}
$$

  - Saturation current (also known as scale current) $I_S = (A_E q D_n n_i^2) / (W N_A)$
    - A strong function of temperature.
    - Proportional to the cross-sectional area of the base-emitter junction.
    - Inverse proportional to the base width W.
- Base current i_B

- i_B is composed of two currents.
  * The holes injected from the base region into the emitter region.

$$i_{B1} = \frac{A_E q D_p n_i^2}{N_D L_p} e^{v_{BE}/V_T} \tag{7.5}$$

  * The holes that have to be supplied by the external circuit due to the recombination.
    · τ_b is the average time for a minority electron to recombine with a majority hole.

$$i_{B2} = \frac{1}{2} \frac{A_E q W n_i^2}{\tau_b N_A} e^{v_{BE}/V_T} \tag{7.6}$$

- Formulation of i_B in terms of i_C.
  * I_S is the saturation current of i_C (refer to Eq.(7.4))
  * $\beta = 1/\left(\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}\right)$ is a constant (normally in the range $50 \sim 200$) for a given transistor.
  * β is mainly influenced by (1) the width of the base region, and (2) the relative dopings of the base region and the emitter region $\frac{N_A}{N_D}$.
    · To achieve high β values, the base should be thin (W small) and lightly doped, and the emitter heavily doped.

$$\begin{aligned} i_B &= i_{B1} + i_{B2} \\ &= I_S\left(\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}\right) e^{v_{BE}/V_T} \\ &= \left(\frac{D_p}{D_n}\frac{N_A}{N_D}\frac{W}{L_p} + \frac{1}{2}\frac{W^2}{D_n \tau_b}\right) i_C \\ &= \frac{1}{\beta} \times i_C \end{aligned} \tag{7.7}$$

- Emitter current i_E
  - From KCL, the i_E and i_C can be related as follows:

$$\begin{aligned} i_E &= i_B + i_C \\ &= \frac{1}{\beta} i_C + i_C \\ &= \frac{1+\beta}{\beta} \times i_C \\ &= \frac{1}{\alpha} \times i_C \\ &= \frac{1}{\alpha} \times I_s e^{v_{BE}/V_T} \end{aligned} \tag{7.8}$$

  * $\alpha = \beta/(1+\beta) \simeq 1$ is a constant for a given transistor.

    * Small change in α corresponds to large changes in β.

- Recapitulation
  - Configuration
    * EBJ (Forward), CBJ (Reverse)
  - Relationship between i_C, i_B, and i_E.
    * $i_C = \beta \times i_B$.
      · β (normally in the range 50∼200) is a constant for a given transistor.
    * $i_C = \alpha \times i_E$.
      · α ($\beta / (1+\beta) \precsim 1$) is a constant for a given transistor.
    * i_B, i_C, and i_E are all controlled by v_{BE}.

$$
\begin{aligned}
i_C &= I_S e^{v_{BE}/V_T} \\
i_B &= \frac{1}{\beta} I_S e^{v_{BE}/V_T} \\
i_E &= \frac{1}{\alpha} I_S e^{v_{BE}/V_T}
\end{aligned} \tag{7.9}
$$

  - Figure 7.9 depicts the large signal equivalent model of the NPN transistor.
    * In Figure 7.9 (a), i_C behaves as a voltage (v_{BE}) controlled current source.

$$
i_C + i_B = i_E = \frac{1}{\alpha} i_C \tag{7.10}
$$

    * In Figure 7.9 (b), i_C behaves as a current (i_E) controlled current source.

$$
\begin{aligned}
& i_C + i_B = i_E \\
\Rightarrow \quad & \alpha i_E + i_B = i_E
\end{aligned} \tag{7.11}
$$

    * The diode D_E represents the forward base-emitter junction.

7.2.2 Reverse Active Mode

- The α and β in the reverse active mode are much lower than those in the forward active mode.
  - α_R is in the range of 0.01 to 0.5.
    * In forward active mode, the collector virtually surrounds the emitter region.
      · Electrons injected into the thin base region are mostly captured by the collector.
    * In reverse active mode, the emitter virtually surrounds the collector region.
      · Electrons injected into the thin base region are partly captured by the

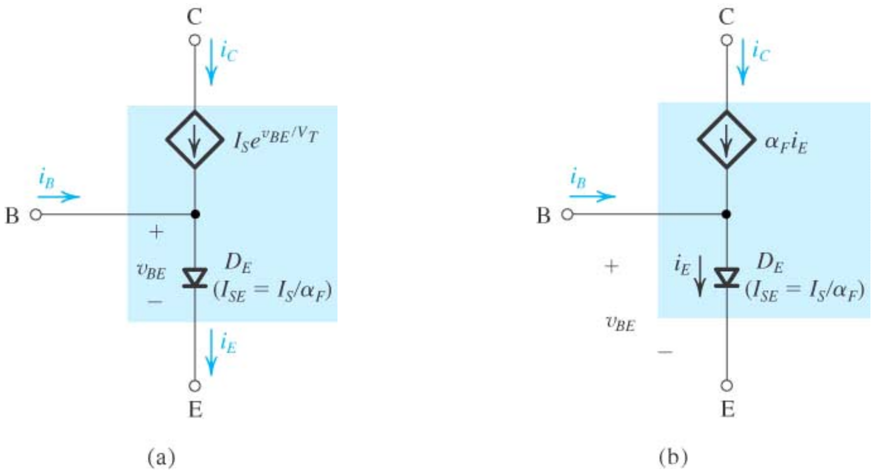

Figure 7.9: Large signal equivalent model of the NPN BJT operating in the forward active mode.

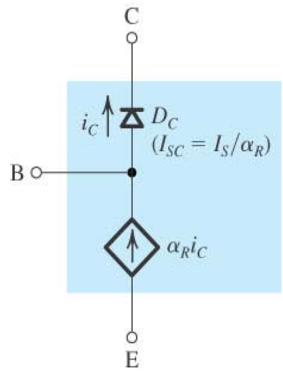

Figure 7.10: Large signal equivalent model of the NPN BJT operating in the reverse active mode.

collector.

- β_R is in the range of 0.01 to 1.

- CBJ has a much larger area than EBJ.
  - The diode D_C denotes the forward base-collector junction.
  - The diode D_C has larger scale current (I_{SC}) than D_E does.
    - The diode D_C has lower voltage drop when forward biased.

7.2.3 Ebers-Moll (EM) Model

- A composite model that can be used to predict the operations of the BJT in all possible modes.
  - Combine Figure 7.9 (b) and Figure 7.10.
- α and β

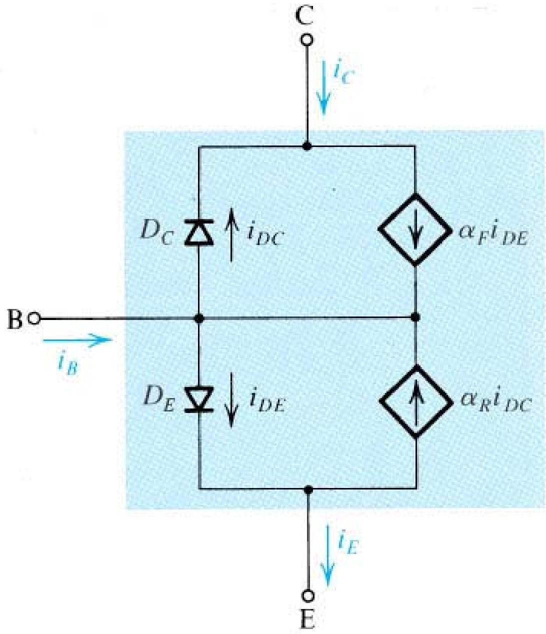

Figure 7.11: Ebers-Moll model of the NPN transistor.

  - α_F and β_F denotes the parameters in forward active mode.
  - α_R and β_R denotes the parameters in reverse active mode.
- Equivalent saturation current I_{SE} and I_{SC}
  - From Figure 7.9 (b) and Figure 7.10, I_{SE} and I_{SC} are the equivalent saturation currents at the EBJ and CBJ, respectively.

$$\begin{aligned} I_{SE} &= \frac{1}{\alpha_F} I_S \\ I_{SC} &= \frac{1}{\alpha_R} I_S \\ \Rightarrow \quad \alpha_F I_{SE} &= \alpha_R I_{SC} = I_S \end{aligned} \tag{7.12}$$

- i_C, i_B, and i_E in the EM model

$$\begin{aligned} i_E &= i_{DE} - \alpha_R i_{DC} \\ i_C &= -i_{DC} + \alpha_F i_{DE} \\ i_B &= (1-\alpha_F) i_{DE} + (1-\alpha_R) i_{DC} \end{aligned} \tag{7.13}$$

  - $i_{DE} = I_{SE}\left(e^{v_{BE}/V_T} - 1\right)$.
  - $i_{DC} = I_{SC}\left(e^{v_{BC}/V_T} - 1\right)$.

- By Eq. (7.12),

$$\begin{aligned} i_E &= \frac{I_S}{\alpha_F}(e^{v_{BE}/V_T}-1) - I_S(e^{v_{BC}/V_T}-1) \\ i_C &= I_S(e^{v_{BE}/V_T}-1) - \frac{I_S}{\alpha_R}(e^{v_{BC}/V_T}-1) \\ i_B &= \frac{I_S}{\beta_F}(e^{v_{BE}/V_T}-1) + \frac{I_S}{\beta_R}(e^{v_{BC}/V_T}-1) \end{aligned} \tag{7.14}$$

  - $\beta_F = \alpha_F/(1-\alpha_F)$.
  - $\beta_R = \alpha_R/(1-\alpha_R)$.

7.2.4 Saturation Mode

- CBJ is in forward bias, i.e., $v_{BC} > 0.4V$.
  - CBJ has larger junction area than EBJ.
    - CBJ has larger saturation current I_S and lower cut-in voltage than EBJ.
    - In forward bias,
      - The voltage drop across CBJ is $0.4V$.
      - The voltage drop across EBJ is $0.7V$.
  - As v_{BC} is increased, i_C will be decreased and eventually reach zero.

$$i_C \simeq I_S e^{v_{BE}/V_T} - \frac{I_S}{\alpha_R} e^{v_{BC}/V_T} \tag{7.15}$$

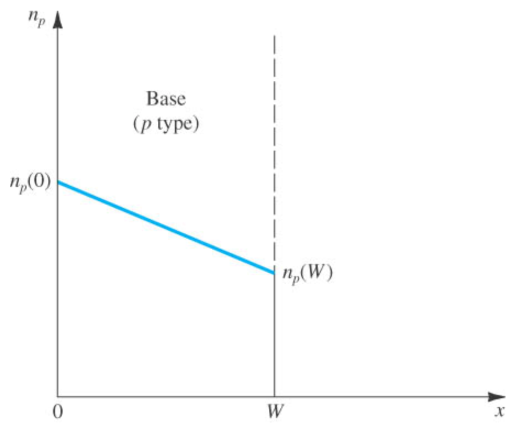

Figure 7.12: Concentration profile of the minority carriers in the base region of an NPN transistor.

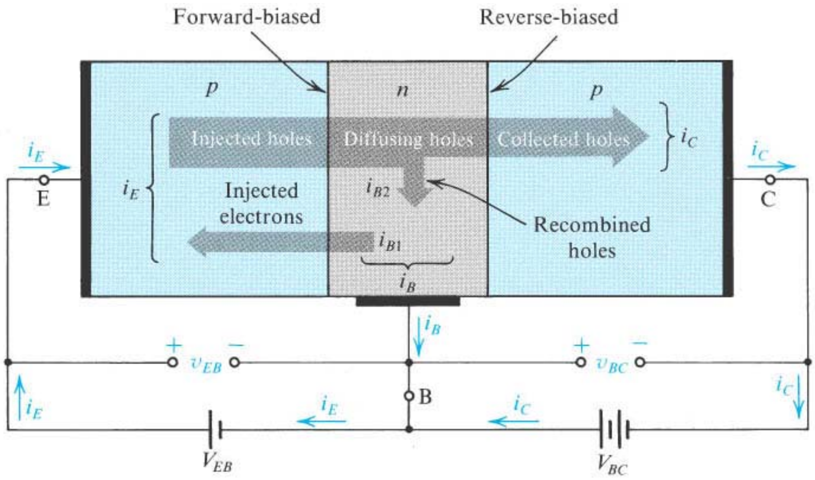

Figure 7.13: Current flow in a PNP transistor biased to operate in the active mode.

7.3 Operations of PNP Transistor

7.3.1 Active Mode

- Current in a PNP transistor is mainly conducted by holes.
- Emitter-Base Junction
  - Forward bias, $v_{EB} > 0$.
  - Holes in the emitter region are injected into the base causing a current i_{E1}.
  - Electrons in the base region are injected into the emitter region causing a current i_{E2}.
    - Generally, $i_{E1} >> i_{E2}$.

$$i_E(t) = i_{E1} + i_{E2} \tag{7.16}$$

- Base region
  - Tapered concentration causes the holes to diffuse through the base region toward the collector.
    - Some of the holes may combine with the electrons.
    - The recombination process is quite small due to lightly doped and thin base region.
- Collector-Base Junction
  - Reverse bias, $v_{BC} > 0$.
  - The holes near the collector side are swept into the collector region causing zero concentration at the collector side.
- Collector current, i_C.
  - Most of the diffusing holes will reach collector region.
    - Only a very small percentage of holes are recombined with the electrons

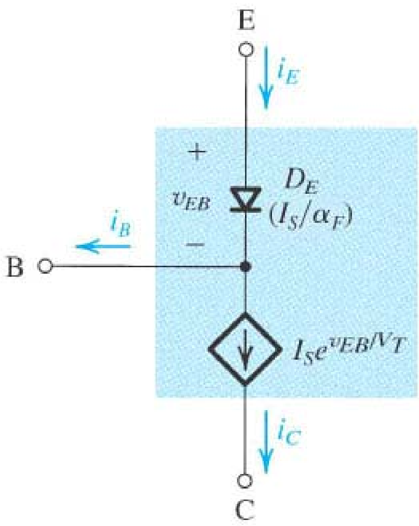

Figure 7.14: Large signal equivalent model of the PNP BJT operating in the forward active mode.

in the base region.

- As long as $v_{BC} > 0$, i_C is independent of v_{BC}.
  - The holes that reach the collector side of the base region will be swept into the collector as collector current.

- Base current i_B
  - i_B is composed of two currents.
    - The electrons injected from the base region into the emitter region.
    - The electrons that have to be supplied by the external circuit due to the recombination.
- Emitter current i_E
  - From KCL, the i_E and i_C can be related as follows:

$$\begin{aligned} i_E &= i_B + i_C \\ &= \frac{1}{\beta} i_C + i_C \\ &= \frac{1+\beta}{\beta} \times i_C \\ &= \frac{1}{\alpha} \times i_C \\ &= \frac{1}{\alpha} \times I_s e^{v_{EB}/V_T} \end{aligned} \tag{7.17}$$

    - $\alpha = \beta / (1 + \beta) \simeq 1$ is a constant for a given transistor.
    - Small change in α corresponds to large changes in β.
- Figure 7.14 depicts the large signal equivalent model of the PNP transistor.

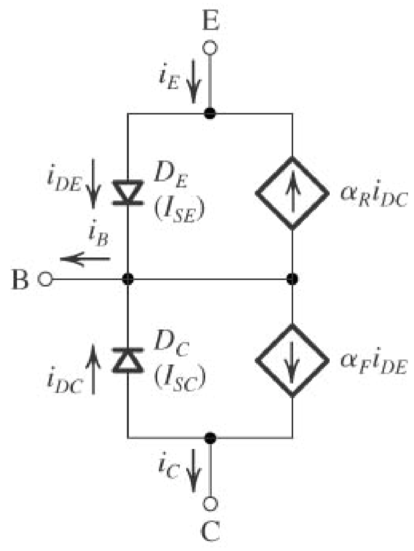

Figure 7.15: Ebers-Moll model of the PNP transistor.

- Figure 7.15 shows the EM model of the NPN transistor.

7.3.2 Reverse Active Mode

- Similar to NPN transistor.

7.3.3 Saturation Mode

- Similar to NPN transistor.

7.3.4 Summary of the i_C, i_B, i_E Relationships in Active Mode

- NPN transistor

$$\begin{aligned} i_c &= I_s e^{v_{BE}/V_T} \\ i_B &= \frac{I_s}{\beta} e^{v_{BE}/V_T} \\ i_E &= \frac{I_s}{\alpha} e^{v_{BE}/V_T} \end{aligned} \tag{7.18}$$

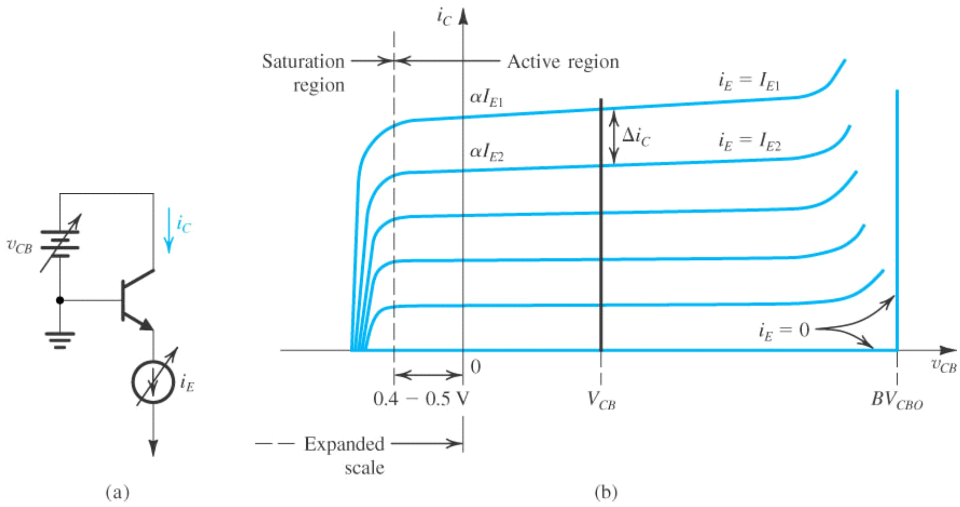

Figure 7.16: The $i_C - v_{CB}$ characteristics of an NPN transistor.

$$
\begin{aligned}
i_C &= \alpha i_E \\
i_C &= \beta i_B \\
i_B &= (1-\alpha) i_E = \frac{i_E}{1+\beta} \\
i_E &= (1+\beta) i_B
\end{aligned}
\tag{7.19}
$$

- PNP transistor.
  - The v_{BE} in Eq. (7.18) is replaced by v_{EB}.

7.4 The $i - v$ Characteristics of NPN Transistor

7.4.1 Common Base ($i_C - v_{CB}$)

- Figure 7.16 depicts the i_C versus v_{CB} for various i_E, which is also known as the common-base characteristics.
  - Input port: emitter and base terminals.
    - Input current i_E.
  - Output port: collector and base terminals.
    - Output current i_C.
  - The base terminal serves as a common terminal to both input port and output port.
- Active Region ($v_{CB} \geq -0.4V$)
  - i_C depends slightly on v_{CB} and shows a small positive slope.

- i_C shows a rapid increase, known as breakdown phenomenon, for a relatively large value of v_{CB}.
- Each $i_C - v_{CB}$ curve intersects the vertical axis at a current level equal to αI_E.
  * Total or large-signal α (common-base current gain)
    · $\alpha = i_C/i_E$, where i_C and i_E denote the total collector and emitter currents, respectively.
  * Incremental or small-signal α
    · $\alpha = \Delta i_C/\Delta i_E$.
  * Usually, the values of incremental and total α differs slightly.

- Saturation Region ($v_{CB} < -0.4V$)
  - CBJ is forward biased.
  - The EM model can be used to determine the v_{CB} at which i_C is zero.

7.4.2 Common Emitter ($i_C - v_{CE}$)

- Figure 7.17 depicts the i_C versus v_{CE} for various v_{BE}, which is also known as the common-emitter characteristics.
  - Input port: base and emitter terminals.
    * Input current i_B.
  - Output port: collector and emitter terminals.
    * Output current i_C.
  - The emitter terminal serves as a common terminal to both input port and output port.
- Active Region ($v_{CB} \geq -0.4V$)
  - i_C increases as the v_{CE} is increased, which is known as Early Effect.
    * At a given v_{BE}, increasing v_{CE} increases the width of the depletion region of the CBJ.
    * The effective base width W is decreased.
    * As shown in Eq. (7.4), I_S is inversely proportional to the base width W.
  - When extrapolated, the characteristics line meet at point on the negative v_{CE} (normally in the range of 50V to 100V), $-V_A$.
    * V_A is a constant for a given transistor.
- Large signal equivalent circuit model in active mode.
  - The linear dependency of i_C on v_{CE} can be formulized as follows:

$$i_C = I_S e^{v_{BE}/V_T}(1 + \frac{v_{CE}}{V_A}) = I_C(1 + \frac{v_{CE}}{V_A}) \tag{7.20}$$

  - The output resistance looking into the collector-emitter terminals.
    * Inversely proportional to the collector current I_C without considering Early effect.

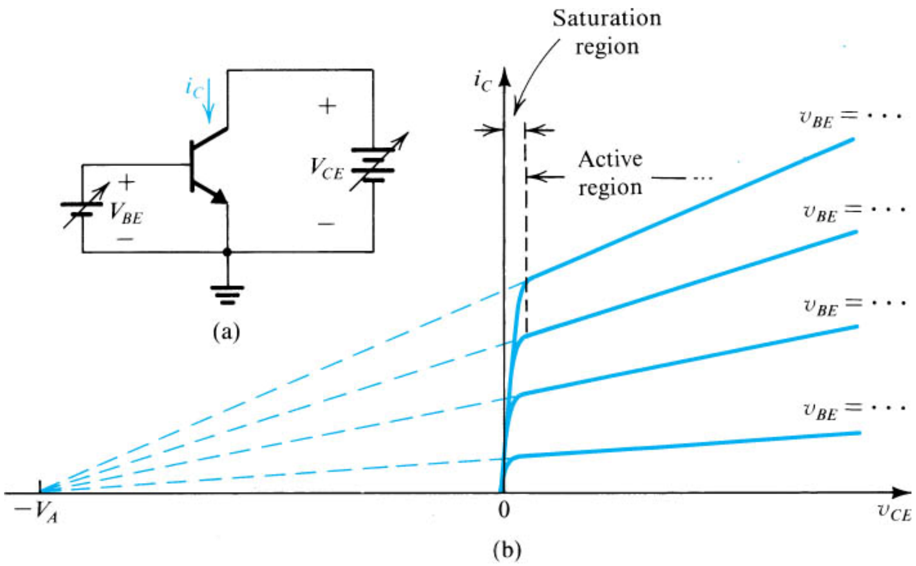

Figure 7.17: The $i_C - v_{CE}$ characteristics of the BJT.

* Controlled by v_{BE}.

$$\Delta i_C = I_S e^{v_{BE}/V_T} (\frac{\Delta v_{CE}}{V_A}) \tag{7.21}$$
$$\Rightarrow \quad r_o = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{V_A}{I_C}$$

– Figure 7.18 depicts the large signal equivalent circuit model of an NPN BJT in the active mode and with the common emitter configuration.
  * Figure 7.18 (a), voltage v_{BE} controls the collector current source.
  * Figure 7.18 (b), the base current i_B controls the collector current source $\beta \times i_B$.

– Large signal or DC β
  * The ratio of total current in the collector to the total current in the base, which represents the ideal current gain (where r_o is not present) of the common-emitter configuration.

$$\beta_{dc} = \frac{i_C}{i_B}|_{v_{CE}=\text{constant}} \tag{7.22}$$

  * β is also known as the common-emitter current gain.

– Incremental or AC β
  * Short-circuit common-emitter current gain.
  * AC β and DC β differ approximately 10% to 20%.

$$\beta_{ac} = \frac{\Delta i_C}{\Delta i_B}|_{v_{CE}=\text{constant}} \tag{7.23}$$

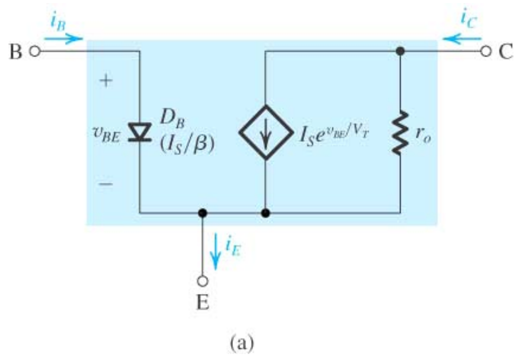

Figure 7.18: Large signal equivalent circuit model of an NPN BJT operating in the active mode and with common-emitter configuration.

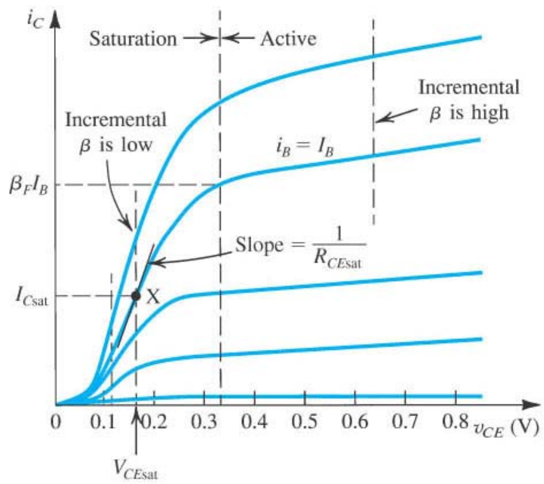

Figure 7.19: An expanded view of the common-emitter characteristic in the saturation region.

- Saturation Region ($v_{CB} < -0.4V$)
  - Figure 7.19 depicts an expanded view of the common-emitter characteristic in the saturation region.
  - Analytical expressions of $i_C - v_{CE}$ using EM model.
    - $v_{BE} = v_{CE} + v_{CB}$.

$$\begin{aligned} i_C &\simeq I_S(e^{v_{BE}/V_T}) - \frac{I_S}{\alpha_R}(e^{v_{BC}/V_T}) \\ I_B &\simeq \frac{I_S}{\beta_F}(e^{v_{BE}/V_T}) + \frac{I_S}{\beta_R}(e^{v_{BC}/V_T}) \end{aligned} \tag{7.24}$$

$$i_C \simeq (\beta_F I_B)\left(\frac{e^{v_{CE}/V_T} - \frac{1}{\alpha_R}}{e^{v_{CE}/V_T} - \frac{\beta_F}{\beta_R}}\right) \tag{7.25}$$

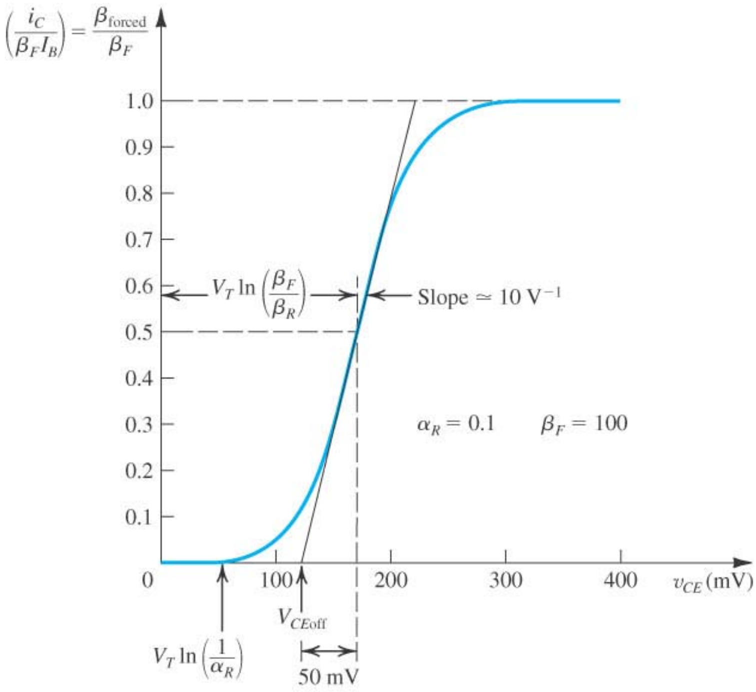

Figure 7.20: Plot of normalized i_C versus v_{CE} for an NPN transistor with $\beta_F = 100$ and $\alpha_R = 0.1$.

- Large signal equivalent circuit model in saturation mode.
  - The saturation transistor exhibits a low collector-to-emitter resistance R_{CEsat}.

$$R_{CEsat} = \frac{\partial v_{CE}}{\partial i_C}|_{i_B=I_B, i_C=I_C} \simeq 1/10\beta_F I_B \tag{7.26}$$

  - At the collector side, the transistor is modeled as a resistance $R_{CE_{sat}}$ in series with a battery v_{CEoff} as shown in Figure 7.21 (c).
    - V_{CEoff} is typically around $0.1V$.
    - V_{CEsat} is typically around $0.1 \sim 0.3V$.

$$V_{CEsat} = V_{CEoff} + I_{Csat} R_{CEsat} \tag{7.27}$$

  - For many applications, the even simpler model shown in Figure 7.21 is used.

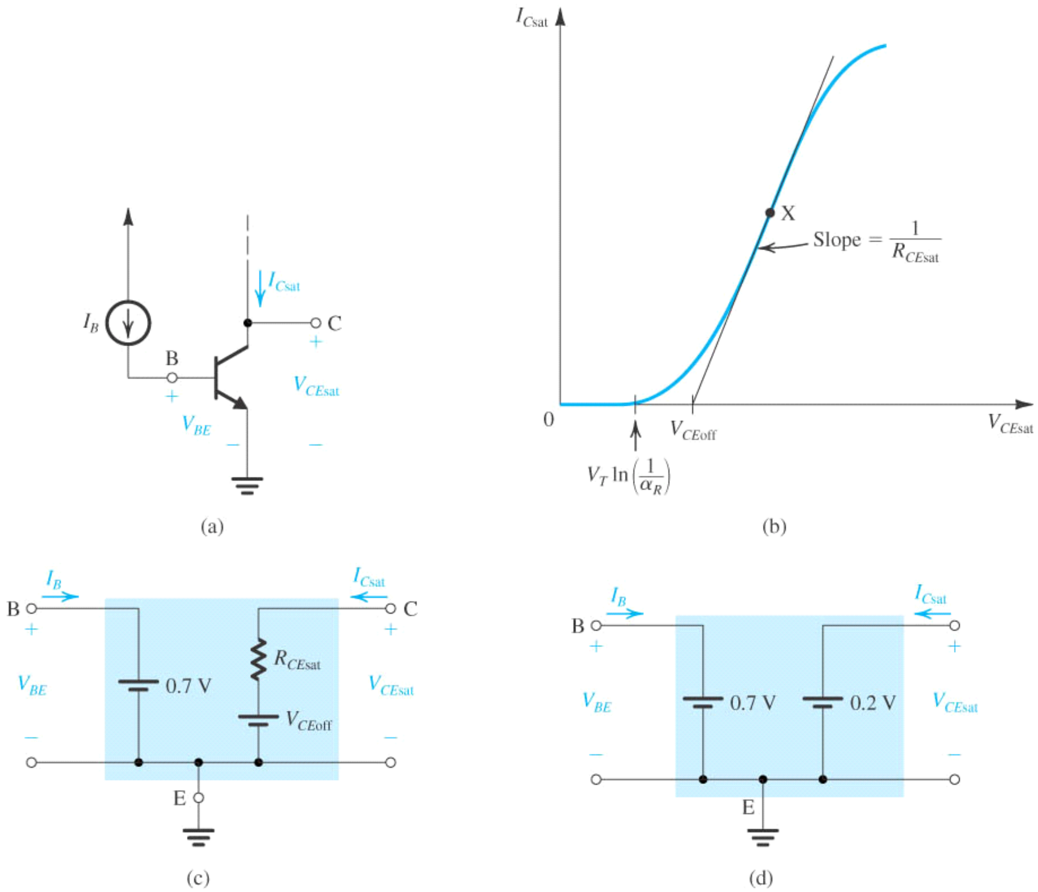

Figure 7.21: Equivalent circuit representation of the saturated transistor.